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# Surface Free Energy of Photon echo on DNA Molecules: Theoretical Study

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**Abstract:** We have study here the topological properties as well as the elastic and thermodynamical properties of DNA molecules from an analysis of an antiferromagnetic spin chain system. The topological property such as the linking number can be derived from the Chem-Simons topology associated with a quantum spin and two indistinguishable fermions are made relatively diverse earlier entanglement, through giving one qubit spin to one of them done the circularly split attractive field that plays the character of shifting the path of quantization axis. Therefore the reverberation arise and the spin must development the transformation of entangled state from symmetric to antisymmetric state finished the closed path. This spin of the attractive field efficiently corresponds near the variation in the path of the flux line. It is exposed that the entanglement of two DNA molecule place spin-echo to one of them marks the transform of Berry phase that can be exact as a analyse of entanglement photon echo technique. If we apply this technique to one spinor previously entanglement by other then the Berry phase is surrounded in the entangled state, consequential the exclusion of dynamical time. This is found to be in good agreement with the thermodynamic entropy as entanglement entropy, charge of encircle a inelastic polymer string bounded by a fine tube.

**Keywords:** DNA Molecule, DNA Polymer Chain, Linking Number, Elastic Forces, Entanglement Entropy

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## 1. Introduction

It is known long ago that quantum mechanics exhibits very peculiar correlation between two physically distant parts of the total system. Two strands of double helix are antiparallel and two polynucleotide chains are coiled about the same axis such that B-DNA (Z-DNA) has right-handed (left-handed) helical sense. The existence of supercoiled DNA has been confirmed in experiments long ago and it was found that in vivo chromosomal DNA molecules contain topological domains along which supercoiling can occur [1-3]. DNA molecules from prokaryotes (cells without nuclear membranes) often adopt the interwound structures which are called "plectonemic" supercoils. In eukaryotes (cells with nuclei and other organelles with their own internal membranes) chromosomal DNA molecules are also known to be organized into topological independent loops [2-5]. Statistical mechanics of supercoiled DNA has been studied by several authors [6].

The experiments of Boles et.al. [7] suggest that thermal

fluctuations determine the structure of supercoils. Experiments of Bednar. et. al [8] indicated that DNA-DNA attraction may compete with fluctuation entropy. In this note we shall study different statistical mechanical aspects of DNA molecules by taking into consideration that a DNA molecules can be viewed as a chain of spin system. In fact as two polynucleotide chains are coiled about the same axis with a specific helical sense in a DNA molecule, we may visualize it such that a spin with a specific orientation is inserted on the axis in the coil such that two adjacent coils have opposite orientations of the spin. This follows from the fact that with each turn two strands move in the opposite side of the axis and so the spin orientations assigned for two adjacent coils should be opposite to each other. In view of this a DNA molecules may be considered to represent an antiferromagnetic chain of spins located on the axis of the supercoil. We shall study the topological properties as well as the elastic and thermodynamical properties of a DNA molecules from an analysis of this spin system and also the entanglement of two DNA molecule

inserting spin-echo to one of them marks the transform of Berry phase that can be exact as a calculate of entanglement. We shall formulate the entangled state formed after spin-echo generate entangled-spin voltage and entangled-inverse spin voltage, produce four-wave combination process is called photon-echo method, through pulse individual separated in period.

## 2. Theoretical Background

We here recapitulate the works on two polynucleotide chains are coiled about the same axis with a specific helical sense in a DNA molecule, this can be viewed as if a spin with a specific orientation is inserted on the axis of the coil such that two adjacent coils have opposite orientations of the spin. In fact with each turn two strands move in the opposite side of the axis and so the spin orientation assigned for the two adjacent coils should be opposite to each other. Thus a DNA molecules may be viewed as a long chain of an antiferromagnetic spin system when the spin is considered to be located on the axis of the supercoil. A unit vector depicting the tangent  $\vec{t} = \partial_s \vec{r}$  where  $\vec{r}(s)$  is a space curve parameterized by the arc length  $s$  can be associated with a spin vector when the spin is located at the spatial point  $x$  on the axis. A spin vector in the Lie algebra of  $SU(2)$  representation can be constructed with bosonic or fermionic oscillators. We write the spin vector  $\vec{S}(x)$  as

$$\vec{S}(x) = \psi_\alpha^\dagger(x) \vec{\sigma}_{\alpha\beta} \psi_\beta(x) \quad (1)$$

where  $\psi^\dagger(\psi)$  is the fermionic oscillator function and  $\vec{\sigma}_{\alpha\beta}$  is the vector of Pauli matrices. A unit vector  $\vec{n}$  is constructed as

$$\vec{n} = (\psi_1^* \psi_2^*) \vec{\sigma} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad (2)$$

with

$$\psi_1 = \left(\cos \frac{\theta}{2}\right) e^{i\phi/2} \quad (3)$$

$$\psi_2 = \left(\sin \frac{\theta}{2}\right) e^{-i\phi/2} \quad (4)$$

We now will study the appearance of Berry phase in the entanglement of two identical spin  $1/2$  quantized particles. The antisymmetric Bell State of two spin  $1/2$  DNA molecules is

$$|\psi_2\rangle = \cos \theta |\psi_1\rangle = \frac{1}{2\pi} (\mathbf{Y} \uparrow - \mathbf{Y} \downarrow) |\psi_1\rangle \quad (5)$$

By the difference of Berry phase factor.

The most general antisymmetric Bell state for two particles A and B situated at the points  $x$  and  $y$  becomes

$$|\psi_2(t)\rangle = (\alpha |\uparrow(t)\rangle |\downarrow(t)\rangle - \beta |\uparrow(t)\rangle |\downarrow(t)\rangle) \quad (6)$$

Where  $\alpha$  and  $\beta$  are two complex coefficients,

With the idea of one DNA molecule rotation of one fermion for a time interval  $\tau$  the spinor comes to its original state acquiring only Berry phase and losing the dynamical phase., We have the new form of the entangle state as

$$|\psi_2(t = \tau)\rangle = (\alpha |\uparrow(t)\rangle |\downarrow(t)\rangle - e^{2iY\tau} |\uparrow(t)\rangle \beta |\downarrow(t)\rangle) \quad (7)$$

As we consider  $\theta = \pi$  the Berry phase is removed along with dynamical phase in the 'spin-echo' method.

This helps us to write

$$\vec{S}(x) = \left(\frac{\sqrt{3}}{2}\right) \psi_\alpha^\dagger(x) \vec{\sigma}_{\alpha\beta} \psi_\beta \quad (8)$$

We can now construct a unit vector  $n_\mu$  with  $\mu = 0, 1, 2, 3$  in 3+1 dimensions incorporating the unit vector  $\vec{n}$  given by eqn. (2)

$$n_\mu = \left(1/\sqrt{2}\right) (\psi_1^* \psi_2^*) \sigma_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad (9)$$

with  $\sigma_0 = I$ ,  $I$  being the identity matrix and  $\vec{\sigma}$  are Pauli matrices. We now construct the topological current

$$J_\mu = \left(1/12\pi^2\right) \epsilon_{\mu\nu\lambda\sigma} \epsilon_{abcd} n_a \partial_\nu n_b \partial_\lambda n_c \partial_\sigma n_d \quad (10)$$

where (a, b, c, d) correspond to (0, 1, 2, 3) and  $(\mu, \nu, \lambda, \sigma)$  correspond to space-time indices. The current  $J_\mu$  can be written in the form [11]

$$J_\mu = \left(1/24\pi^2\right) \epsilon_{\mu\nu\lambda\sigma} Tr(g^{-1} \partial_\nu g)(g^{-1} \partial_\lambda g)(g^{-1} \partial_\sigma g) \quad (11)$$

with  $g = n_0 I + i\vec{n} \cdot \vec{\sigma}$  which belongs to the group  $SU(2)$ . If we now demand that in Euclidean 4-dimensional space-time the field strength  $F_{\mu\nu}$  of a gauge potential  $A_\mu$  vanishes at all points on the boundary  $S^3$  of a certain volume  $V^4$  inside which  $F_{\mu\nu} \neq 0$  the gauge potential tends to a pure gauge towards the boundary and we write

$$A_\mu = g^{-1} \partial_\mu g \quad (12)$$

with  $g \in SU(2)$ .

We can now write the topological current given by (11) as [10]

$$J_\mu = \left(1/16\pi^2\right) \epsilon^{\mu\nu\lambda\sigma} Tr\{A_\nu F_{\lambda\sigma} + (2/3) A_\nu A_\lambda A_\sigma\} \quad (13)$$

with  $A_\mu$  given by eqn.(8). It is noted that as the spin vector is constructed from the unit vector  $\vec{n}$  given by (2) which is incorporated in the current  $J_\mu$  as is evident from eqn. (10),

we can associate spin with this current  $J_\mu$ . In fact we can consider the topological Lagrangian in terms of the  $SU(2)$  gauge fields in affine space

$$L = -(1/4) \text{Tr} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \quad (14)$$

Now the gauge connection associated with the Lagrangian in this equation

$$L_{eff}^\dagger = -(i/2)(\dot{\xi} - \dot{\phi} \cos \theta) \quad (15)$$

due to any change in  $\theta$ ,  $\phi$ ,  $\xi$  resulting a gauge transformation, this equation giving rise to Berry phase.

Now the necessary geometrical phase of the only quantized spinor

$$Y^\dagger = i \int L_{eff}^\dagger dt = i \oint A^\dagger(\lambda) d\lambda \quad (16)$$

This gives rise to the topological current [12]

$$\vec{J}_\mu = \varepsilon^{\mu\nu\lambda\sigma} \vec{a}_\nu \times \vec{f}_{\lambda\sigma} = \varepsilon^{\mu\nu\lambda\sigma} \partial_\nu \vec{f}_{\lambda\sigma} \quad (17)$$

where we have taken the  $SU(2)$  gauge field  $A_\mu$  and corresponding field strength  $F_{\mu\nu}$  as

$$A_\mu = a_\mu \cdot \vec{\sigma} \quad \text{and} \quad F_{\mu\nu} = f_{\mu\nu} \cdot \vec{\sigma} \quad (18)$$

$\vec{\sigma}$  being vector of Pauli matrices. From this it appears that the spin vector  $\vec{S}(x)$  can be depicted as the topological current  $\vec{J}_\mu$  given by eqn.(17). In terms of this current a spin system on a lattice can be viewed as if currents are located on the vertices when gauge fields lie on links [9]. This helps us to consider the spin system associated with a DNA supercoil in terms of the Chern-Simons topology as will be discussed in the next section. Now the necessary geometrical phase of the only quantized spinor eqn. (16),

This shows that for quantized spinor the Berry Phase is a solid angle subtended about the quantization axis. For  $\theta = 0$  the minimum value of  $Y^\dagger$  is 0 and  $\theta = \pi$  maximum.

Spin up case we have

$$\Theta^\dagger(\lambda) = (1/2)(1 - \cos \theta) \quad (19)$$

This leads to have the noise dependent Berry Connection of the quantized spinor

$$\Theta^\dagger(\lambda) = (1/2)(1 - \cos \theta + \sin \theta \delta\theta) \quad (20)$$

Now the result a modification of Berry phase

$$\Pi^\dagger = \pi(1 - \cos \theta + \sin \theta \delta\theta) \quad (21)$$

And similar for down spinor

$$\Pi^\downarrow = \pi(1 - \cos \theta - \sin \theta \delta\theta) \quad (22)$$

Where we consider  $\Pi^\uparrow, \Pi^\downarrow$  as the noise induced Berry phase for the spin up and spin down quantized practices in that order [13-15]. Now the entangled state of two identical spinor, as we find in eqn.(5), that the evolution of the state at a exacting instant depends on the distinction of  $Y^\uparrow$  and  $Y^\downarrow$  which implies boost of noise by twice. The effect of noise in the entangled state formed after 'spin-echo' will be less as realized from eqn.(7). Now the entangled state formed after spin-echo generate entangled-spin voltage (ESV).

$$\mathfrak{S}_{ESV} = Y^\uparrow - Y^\downarrow \quad (23)$$

and entangled-inverse spin voltage (EISV)

$$\mathfrak{S}_{EISV} = Y^\downarrow - Y^\uparrow \quad (24)$$

$Y^\uparrow$  and  $Y^\downarrow$  and are the entangled state of two identical up and down spinor.

Now the spinor produce four-wave entangled-spin voltage (ESV) and entangled-inverse spin voltage (EISV) produce photon-echo.

$$\mathfrak{S}_{ESV} + \mathfrak{S}_{EISV} = Y^\downarrow - Y^\uparrow + Y^\uparrow - Y^\downarrow = 0 \quad (25)$$

$$\Rightarrow \mathfrak{S}_{ESV} + \mathfrak{S}_{EISV} = 0 \quad (26)$$

$$\Rightarrow \mathfrak{S}_{ESV} = -\mathfrak{S}_{EISV}$$

### 3. Conclusion

The present analysis suggests that quantum framework of DNA molecules in terms of an antiferromagnetic spin chain gives rise to the entanglement entropy which induces the entropic potential associated with the free energy per length corresponding to the entropy cost of confining a stiff polymer inside a narrow tube. The entanglement entropy effectively represents the thermodynamic entropy and this repulsive entropic potential opposes the elastically driven collapse of a supercoil which can occur at zero temperature. The entanglement entropy has a very significant implication in that it measures the total information content in the system. Indeed when a DNA is regarded as a linear repository of sequence information the entanglement can be used to transcribe information and entropy determines the quantity of information which is available for transcription. In view of this a measure of this entanglement entropy can be taken to determine the quantity of genetic information which can be transcribed. An attractive substitute to a single DNA molecular system approach represent a photon-echo quantum memory [16]. Clearly, with quantum state transport among a photon and an ensemble of inhomogeneously broadened molecules absorbers increases the difficulty of the structure. We can also may say that here the noise is accountable for the fluctuation of quantization that can be practical for the entanglement of Quantum Hall particles in the non-plateau and plateau area.

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